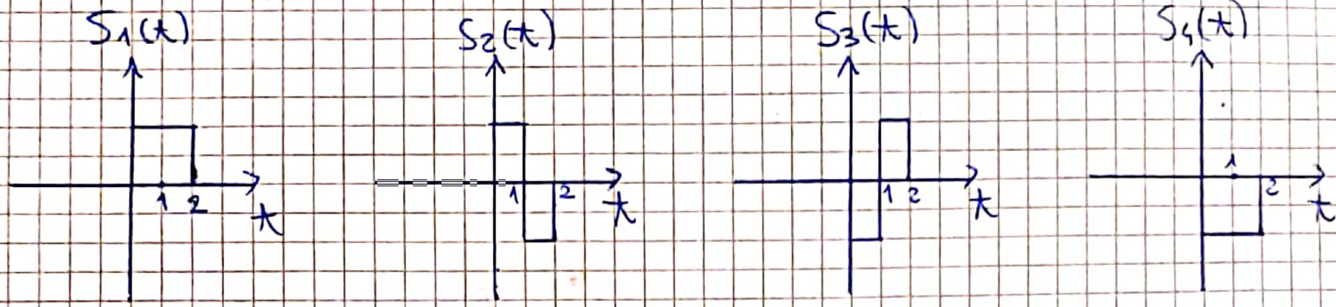
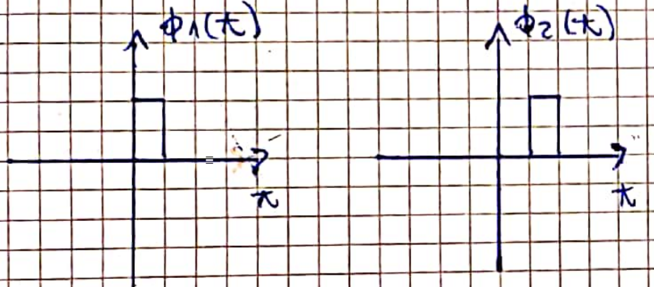
Esercizio (spazio segnali)



Ho un set di 4 segnali.
Qual è il minimo $N \leq M$ per rappresentare i segnali sopra nello spazio dei segnali?



← questi bastano in quanto:

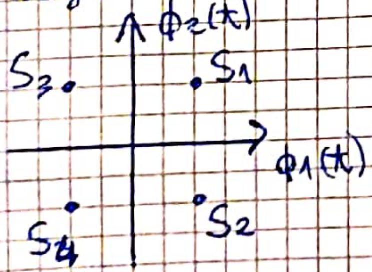
$$S_1(t) = C_{11}\phi_1(t) + C_{12}\phi_2(t)$$ ← $S_1(t) \Leftrightarrow (1,1)$

$$C_{11}(t) \triangleq \int S_1(t)\,\phi_1(t)\,dt = 1$$
$$C_{12}(t) \triangleq \int S_1(t)\,\phi_2(t)\,dt = 1$$

METODO ANALITICO PER TROVARE COEFFICIENTI, MA POSSO ANCHE INTUIRLI DALLE FIGURE

$S_2(t) \Leftrightarrow (1,-1)$
$S_3(t) \Leftrightarrow (-1,1)$
$S_4(t) \Leftrightarrow (-1,-1)$

Ora che ho i coefficienti posso rappresentarli nello spazio dei segnali:



← COSTELLAZIONE OF M-ORDER MODULATION ← $N = 2$

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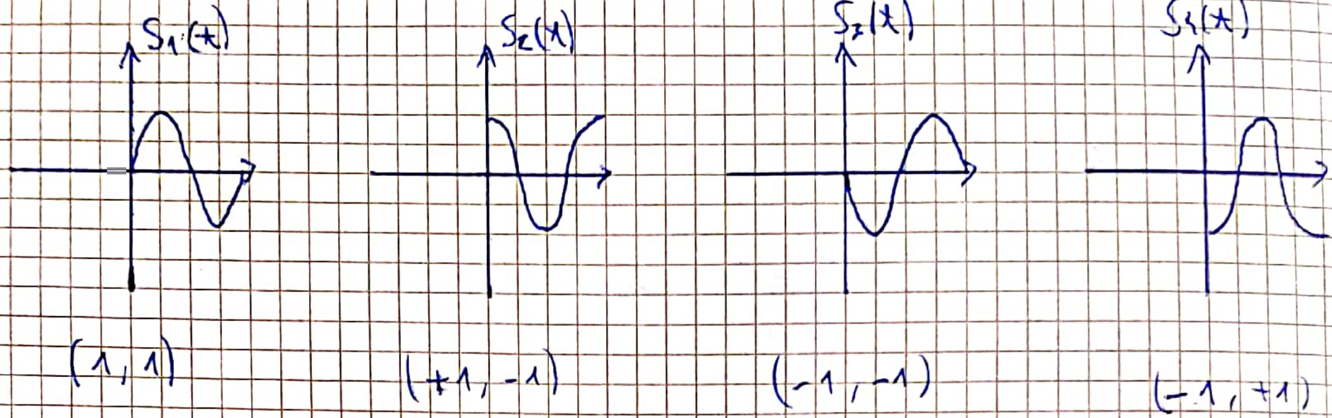
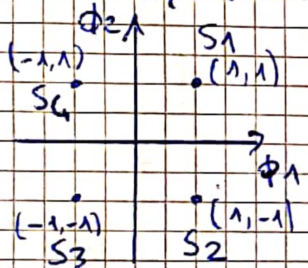
Esercizio (QPSK)

QUADRATURE ← $M=4$

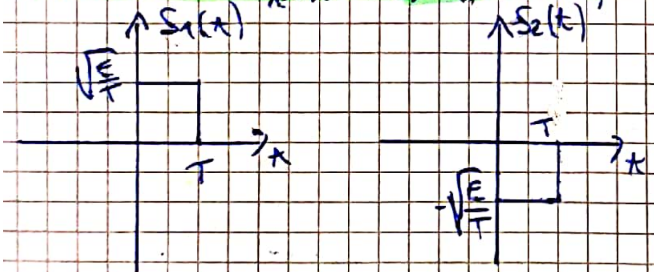
$$S_i(t) = \pm\sqrt{\frac{2}{T}}\cos(2\pi f_c t) \pm \sqrt{\frac{2}{T}}\sin(2\pi f_c t) = C_1\Phi_1 + C_2\Phi_2$$



Con differenti rappresentazioni ho la stessa rappresentazione nello spazio dei segnali



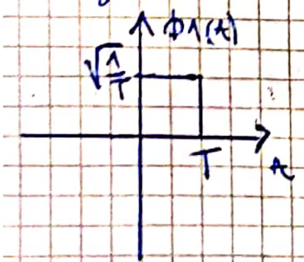
Esercizio (2-PAM) → ANTIPODAL MODULATION



ENERGIA

$$\int_0^T |S_1^2(t)|\,dt = \int_0^T \frac{E}{T}\,dt = \frac{E}{N}$$

Di quanti segnali ho bisogno per rappresentare nello spazio dei segnali? = solo 1



$N = 1$

Costellazione 2-PAM

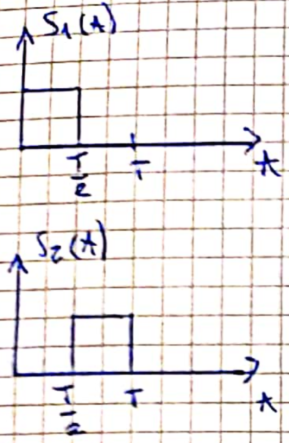
← rappresento con 1 retta

$$S_1(t) = C_1 \cdot \Phi_1(t) \Rightarrow C_1 = \sqrt{E} \qquad C_1 = \int_0^T S_1(t)\,\Phi_1(t)\,dt = \sqrt{E}$$

$$S_2(t) = C_2 \cdot \Phi_1(t) \Rightarrow C_2 = -\sqrt{E}$$

Scanned by CamScanner

Esercizio (2-PAM)

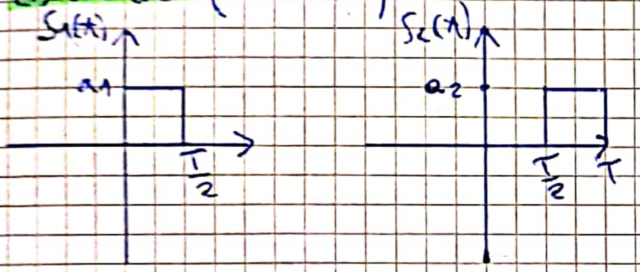


→ ortogonali → devo usare waveform per rappresentare 2-dimensioni

↓

piano costellation

Esercizio (2-PPM)

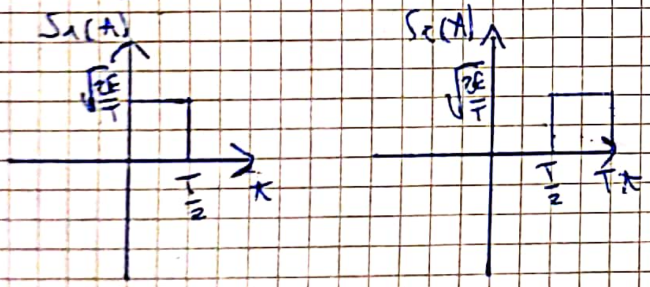


$$E_1 = \int_0^{\frac{T}{2}} a_1^2 \, dt = \frac{a_1^2}{2} T$$

$$a_1 = \sqrt{\frac{2E_1}{T}}$$

$$a_2 = \sqrt{\frac{2E_2}{T}}$$

In genere si trasmettono due simboli con uguale E

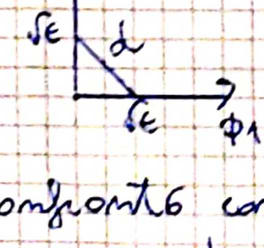


$$\left.\begin{array}{l} \phi_1 = \dfrac{s_1(t)}{\sqrt{E}} \\[2ex] \phi_2 = \dfrac{s_2(t)}{\sqrt{E}} \end{array}\right\} \text{BASIS SIGNAL SET}$$

Sono ortogonali $\int s_1 \cdot s_2 \, dt = 0$

$$s_1(t) = \sqrt{E}\,\phi_1(t) + 0 \cdot \phi_2(t) \Rightarrow s_1(t) = (\sqrt{E}, 0)$$

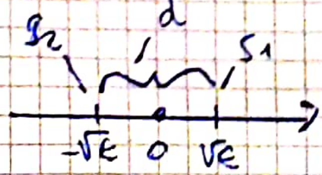
$$s_2(t) = 0 \cdot \phi_1(t) + \sqrt{E} \cdot \phi_2(t) \Rightarrow s_2(t) = (0, \sqrt{E})$$



$$d = \sqrt{E + E} = \sqrt{2E}$$

← COSTELLAZIONE 2-PPM

confronto con antipodal (2-PAM)



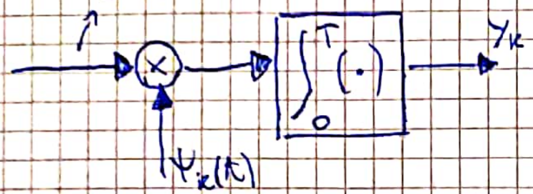
→ Same energy, but higher distance with 2-PAM

2-PAM >> 2-PPM ⇒ più robusta al rumore per via di maggiore distanza

Esercizio (correlator)

$$S_m(t) = \sum_{j=1}^{N} s_j \psi_j(t)$$

$$S_m(t) + n(t)$$



$$y_k = \int_0^T \left[S_m(t) + n(t)\right] \psi_k(t)\, dt =$$

$$= \sum_{j=1}^{N} s_j \int_0^T \psi_k(t)\psi_j(t)\, dt + \int_0^T n(t)\psi_k(t)\, dt$$

$$y_k = s_k + n_k \Rightarrow$$ Così ricavo val medio e varianza

$$n_k = \int_0^T n(t)\psi_k\, dt \Rightarrow E[n_k] = \int_0^T E[n(t)]\, \psi_k(t)\, dt = 0$$

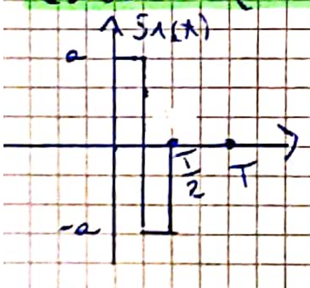
$$\sigma^2 = E[n_k^2] = E\left[\int_0^T\int_0^T n(\tau)n(\nu)\psi_k(\tau)\psi_k(\nu)\, d\tau\, d\nu\right] =$$

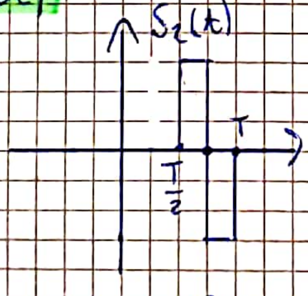
$$= \int_0^T\int_0^T \underbrace{E(n(\tau)n(\nu))}_{\text{CROSS CORRELATION}} \psi_k(\tau)\psi_k(\nu)\, d\tau\, d\nu = \frac{N_0}{2}\int_0^T \underbrace{\psi_k^2(\tau)\, d\tau}_{1 \text{ (ortonormali)}} = \frac{N_0}{2}$$

$$E[y_k] = s_k$$

$$\sigma^2 = \frac{N_0}{2}$$

Esercizio (correlator)

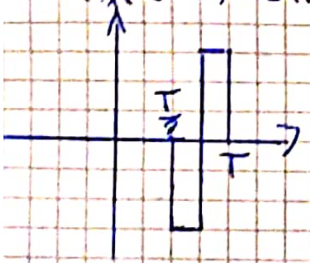


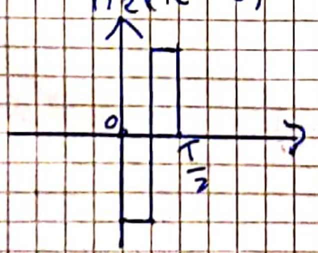


→ ortogonali → $N = 2$

Faccio time reverse e shift e poi pongo $t = T$ (shift a destra)

$h_1(t-\tau) = s_1(t-\tau)$ $\qquad$ $h_2(t-\tau) = s_2(t-\tau)$ ← in questo caso $t = T$





① Trasmetto $S_1(t)$ e calcolo output

$\left.\begin{matrix} m_1? \\ \sigma_1^2? \end{matrix}\right] Y_1 \quad m_1 = E_1 = \int_0^T |S_1(t)|^2 dt = \frac{a^2T}{2}; \quad \sigma_1^2 = \frac{N_0}{2}\int_0^T S_1^2(t)\,dt = \frac{E_1 N_0}{2}$

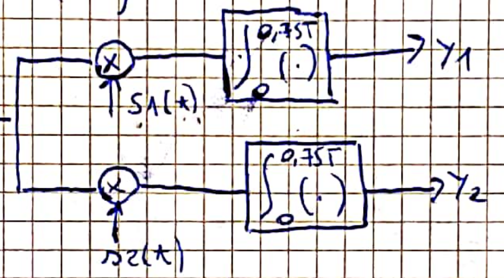
$\left.\begin{matrix} m_2? \\ \sigma_2^2? \end{matrix}\right] Y_2 \quad m_2 = E[Y_2] = S_2 = 0; \quad \sigma_2^2 = E_2 \cdot \frac{N_0}{2}$

$\left(S_2 = \int s_1(\tau)\, h_2(T-\tau)\, d\tau = \int_{-\infty}^{+\infty} s_1(\tau)\, s_2(T-T+\tau)\, d\tau = \int_{-\infty}^{+\infty} s_1(\tau) s_2(\tau)\, d\tau = 0 \right)$

|| 0 perché ortogonali

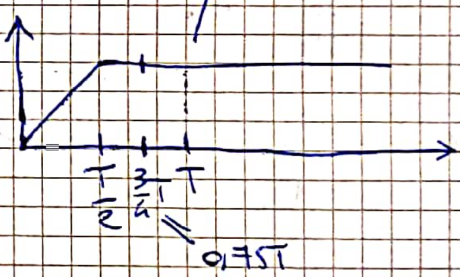
In Y_2 abbiamo in output solo noise
In Y_1 abbiamo in output signal + noise
Il detector fa $Y_1 > Y_2$ e se è vero abbiamo trasmesso S_1, sennò S_2 (esempio)

② $s_1(t) + n(t)$

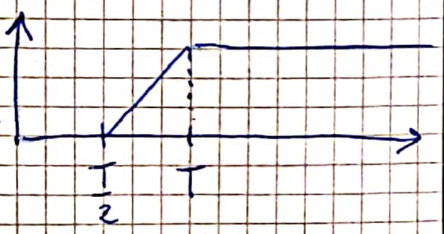


Se faccio sampling in T (se corr. ideale) o se lo faccio poco prima o dopo, in questo caso è indifferente

$\int_0^t S_1^2(t)\,dt = \begin{cases} a^2 t & t < \frac{T}{2} \\ \frac{a^2T}{2} & \frac{T}{2} < t < T \end{cases}$



$\int_0^t S_2^2(t)\,dt = \int_0^{T/2} \underbrace{\dots}_{0} + \int_{T/2}^{t} a^2\, dt = a^2\left(t - \frac{T}{2}\right)$



$m_2 = a^2\left(\frac{3}{4}T - \frac{T}{2}\right) = \frac{a^2T}{4}$ ← metà di prima

$\sigma_2^2 = \frac{N_0}{2}\int_0^{\frac{3}{4}T} s^2(t)\,dt = \frac{a^2TN_0}{8} = \frac{N_0}{2}\frac{a^2T}{4}$

In questo caso se campiono un pò dopo o prima cambia.
Se campiono in 0,75T ottengo metà dell'ideal correlator

③ Calcolo SNR

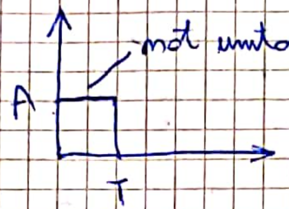
$SNR = \frac{\left(\frac{E_2}{2}\right)^2}{\frac{N_0}{2}\frac{E_2}{2}} = \frac{\left(\frac{E_2^2}{4}\right)}{\frac{N_0}{4}E_2} = \frac{E_2}{N_0}$

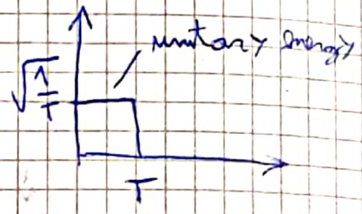
Se avessi trasmesso S_2 avrei perso metà dell'SNR e in dB avrei perso 3 dB. → Se correlator non ideale → uso detector unbalanced

Esercizio 6 (ON-OFF SIGNALING)

binary trasmission

$$\begin{cases} 0 & S_0 = 0 \\ 1 & S_1 = A \quad 0 \le t \le T \end{cases}$$



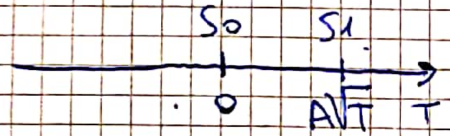


$\mathcal{E} = x^2 \cdot T = 1$

$x = \sqrt{\frac{1}{T}}$

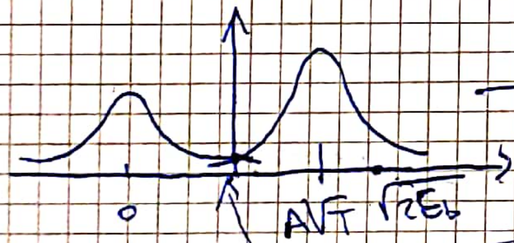
$\psi_1 = S_1 \cdot \sqrt{\frac{1}{T}} = A \Rightarrow$ da qui capisco che valore deve avere S_1

$S_1 = A\sqrt{T}$



$$\mathcal{E}_{AV} = \frac{1}{2}\mathcal{E}_{S_0} + \frac{1}{2}\mathcal{E}_{S_1} = \frac{1}{2}\mathcal{E}_{S_1} = \frac{A^2T}{2} \Rightarrow A\sqrt{T} = \sqrt{2\mathcal{E}_b}$$

energy per symbol in general → in questo caso energy per bit



→ scelgo threshold in $0 = \alpha$

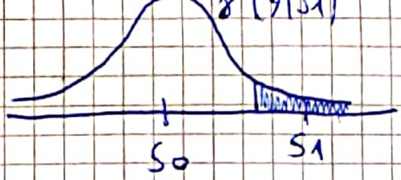
calcolo P_{error} →
$$\begin{cases} f(r|S_0) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{r^2}{N_0}} \\ f(r|S_1) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-A\sqrt{T})^2}{N_0}} \end{cases}$$

li uguaglio e ottengo $r^2 - A\sqrt{T} - 2Ar\sqrt{T} + A^2T = r^2 \Rightarrow r = \frac{A\sqrt{T}}{2}$
quindi intuitivamente avevamo messo bene $\alpha = 0$ che corrisponde a livello di energia ad $\frac{A\sqrt{T}}{2}$

$$P_e = \frac{1}{2}P(S_1|S_0) + \frac{1}{2}P(S_0|S_1)$$

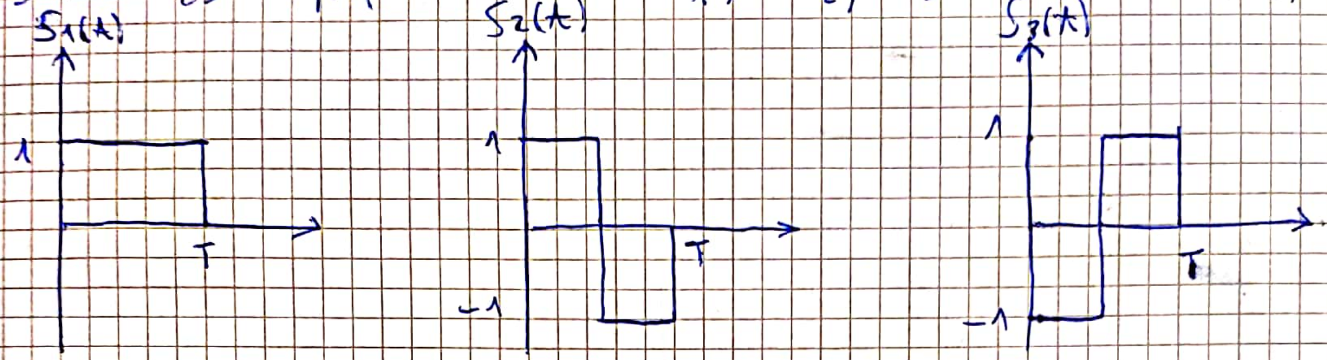
$$P(S_1|S_0) = \frac{1}{\sqrt{\pi N_0}}\int_{\frac{A\sqrt{T}}{2}}^{+\infty} e^{-\frac{r^2}{N_0}}\,dr = \frac{1}{\sqrt{\pi N_0}}\int_{\sqrt{\frac{\mathcal{E}_b}{N_0}}}^{+\infty} e^{-\frac{t^2}{2}}\cdot\sqrt{\frac{N_0}{2}}\,dt = \underbrace{\frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{\mathcal{E}_b}{N_0}}}^{+\infty} e^{-\frac{t^2}{2}}\,dt}_{\text{Q FUNCTION}}$$

$P_{error} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$ $\qquad \frac{A\sqrt{T}}{2} \to \frac{\sqrt{E_b}}{N_0}$

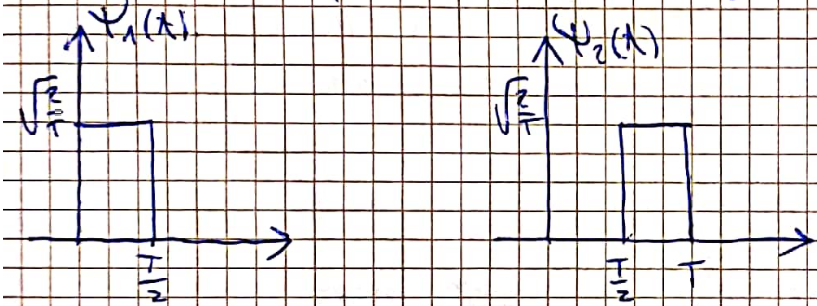


Esercizio 6 (MINIMUM DISTANCE CRITERIA)

3 Messaggi equiprobabili m_1, m_2, m_3 $\quad P(m_1)=P(m_2)=P(m_3)=\frac{1}{3}$



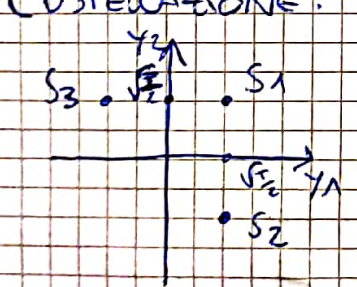
Serve N=2 per rappresentare gli impulsi



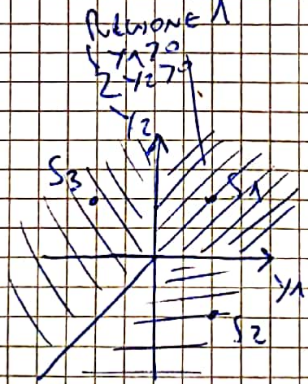
$$\Rightarrow \begin{array}{l} S_1\left(\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right) \\ S_2\left(\sqrt{\frac{T}{2}}, -\sqrt{\frac{T}{2}}\right) \\ S_3\left(-\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right) \end{array}$$

Ho scelto queste coordinate perché se moltiplico per $\varphi_1(t)$ ottengo l'impulso iniziale.

COSTELLAZIONE:



$\Rightarrow$



$\leftarrow$ A seconda della regione scelgo S_1, S_2 o S_3

Ho bisogno di 2 Matched filter perché ho 2 φ

$P_{errore} = \frac{1}{3}P(e|S_1) + \frac{1}{3}P(e|S_2) + \frac{1}{3}P(e|S_3)$

$P(e|S_1) = 1 - P_{correct}(S_1)$

$P_{correct}(S_1) = \underbrace{P(Y_1>0)\cdot P(Y_2>0)}_{\text{JOINT PROBABILITY}} \to$ eventi indipendenti, che quindi posso fattorizzare.

$P(Y_2>0)$

$P(Y_1>0) = P\left(\sqrt{\frac{T}{2}} + n_1 > 0\right) = P\left(n_1 > -\sqrt{\frac{T}{2}}\right) = \frac{1}{\sqrt{\pi N_0}}\int_{-\sqrt{\frac{T}{2}}}^{+\infty} e^{-\frac{x^2}{N_0}}dx$

SOSTITUZIONE $\boxed{\frac{x^2}{N_0} = \frac{t^2}{2}}$

$= \frac{1}{\sqrt{2\pi}}\int_{-\sqrt{\frac{T}{N_0}}}^{+\infty} e^{-\frac{t^2}{2}}dt = 1 - Q\left(\sqrt{\frac{T}{N_0}}\right)$

$$P_e(S_1) = \left[1 - Q\left(\sqrt{\frac{T}{N_0}}\right)\right]^2 = \left[1 + \underbrace{Q^2\left(\sqrt{\frac{T}{N_0}}\right)}_{\simeq 0} - 2Q\left(\sqrt{\frac{T}{N_0}}\right)\right] = 1 - P_{e\,\text{correct}}(S_1) = 2Q\left(\sqrt{\frac{T}{N_0}}\right)$$

$$P_{\text{CORRECT}}(S_2) = P(y_2 < 0,\ y_1 > y_2) = ?$$

$$P(y_2 < 0) = P\left(-\sqrt{\frac{T}{2}} + n_2 < 0\right) = P\left(n_2 < \sqrt{\frac{T}{2}}\right) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\sqrt{\frac{T}{2}}} e^{-\frac{x^2}{N_0}}\, dx = 1 - Q\left(\sqrt{\frac{T}{N_0}}\right)$$

$$P(y_1 > y_2) = P\left(\sqrt{\frac{T}{2}} + n_1 - n_2 + \sqrt{\frac{T}{2}} > 0\right) = P\left(n_1 - n_2 > -2\sqrt{\frac{T}{2}}\right) =$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_{-2\sqrt{\frac{T}{2}}}^{+\infty} e^{-\frac{x^2}{2N_0}}\, dx = 1 - Q\left(\sqrt{\frac{2T}{N_0}}\right)$$

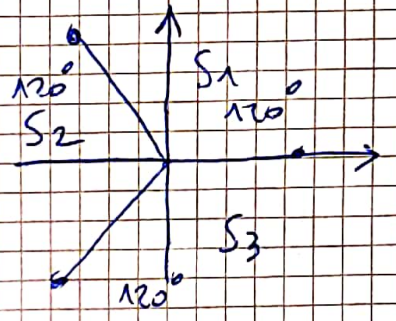
$$P_{\text{correct}}(S_2) = \left[1 - Q\left(\sqrt{\frac{T}{N_0}}\right)\right]\left[1 - Q\left(\sqrt{\frac{2T}{N_0}}\right)\right]$$

$$P_{\text{error}}(S_2) = 1 - P_{\text{correct}}(S_2) \leq Q\left(\sqrt{\frac{T}{N_0}}\right) + Q\left(\sqrt{\frac{2T}{N_0}}\right)$$

$\Rightarrow$ S_1 meno robusto rispetto a noise $\leftarrow$ perché regione piccola

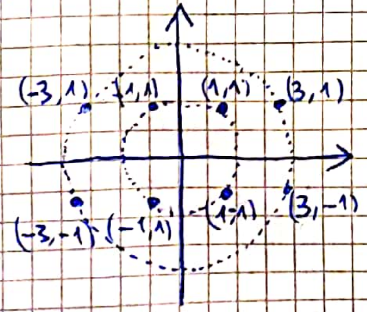
$P_{err}(S_2) = P_{err}(S_3)$

In questo modo posso far sì che le regioni siano robuste allo stesso modo:



$\leftarrow$ PSK $\leftarrow$ meno + robusto S_1 e
− robusti S_2 e S_3
$\shortparallel$
TRADEOFF

Esercizio 8-QAM



$\leftarrow$ distanza max tra due punti è 2

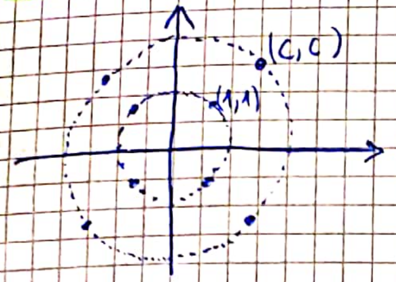
Nel cerchio piccolo ho bisogno della stessa energia per trasmettere ogni simbolo

$$\mathcal{E}_1 = d^2 = (\sqrt{2})^2 = 2$$

Nel cerchio grande lo stesso $\mathcal{E}_2 = d^2 = (\sqrt{9+1})^2 = 10$

$$\mathcal{E}_{AV} = \frac{1}{8}(4 \cdot 2 + 4 \cdot 10) = 6$$

Esercizio 8-QAM



$d_{min} = 2$ tra due punti

$c = \sqrt{3+2\sqrt{2}}$

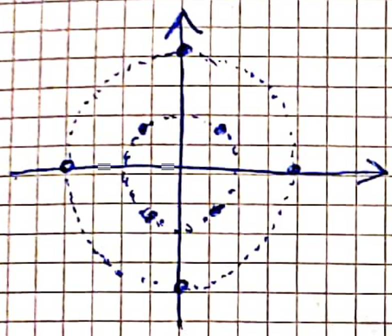
$d_c = \sqrt{3+2\sqrt{2}+3+2\sqrt{2}} = \sqrt{6+4\sqrt{2}} \Rightarrow d_c^2 = 6+4\sqrt{2}$

$\mathcal{E}_c = 6+4\sqrt{2}$

$\mathcal{E}_1 = 2$

$\mathcal{E}_{AV} = \frac{1}{8}\left(4 \cdot 2 + 4\left(6+4\sqrt{2}\right)\right) = \frac{1}{8}\left(8+24+16\sqrt{2}\right) = 6{,}8$

Esercizio 8-QAM

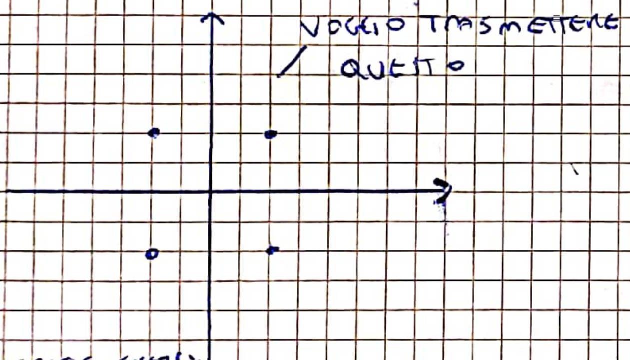


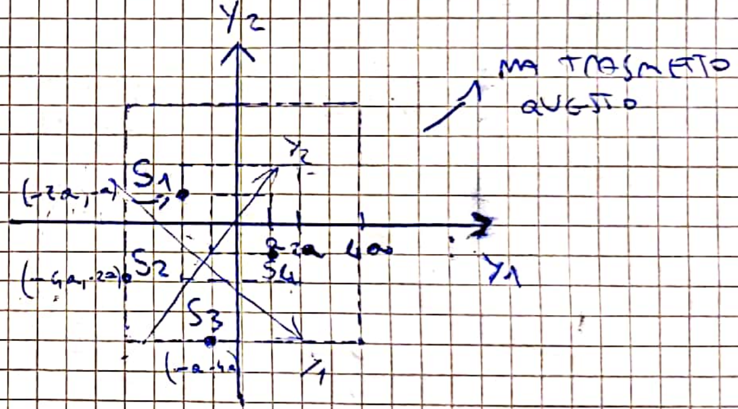
$d_{min} = 2$ tra due punti

- best choice for simplicity of representation
- optimal costellation, not the letter

$\rightarrow$ rectangular representation

Esercizio Q-PSK





AVERAGE ENERGY TRASMITTED PER SYMBOL

$\mathcal{E}_{AVG} = \frac{1}{4}\left(\mathcal{E}_{S1} + \mathcal{E}_{S2} + \mathcal{E}_{S3} + \mathcal{E}_{S4}\right) = \frac{1}{4} a^2 \cdot (5+20+2+17) = 11a^2$

$= a = \sqrt{\frac{\mathcal{E}_S}{11}}$

$P_{errore} = Q\left(\sqrt{\frac{d^2}{2N_0}}\right) \leftarrow$ DEVO CALCOLARE OPTIMAL RECEIVER

CASO IDEAL QPSK $\leftarrow$ CONOSCO ESATTAMENTE LA POSIZIONE DI $S_1 S_2 S_3 S_4$

$d^2 = (-2a-a)^2 + (a+a)^2 = 13a^2$

$\hookrightarrow$ DISTANZA TRA S_1 E S_4

$P_{error} = Q\left(\sqrt{\frac{13a^2}{2N_0}}\right) = Q\left(\sqrt{\frac{13\,\mathcal{E}_S}{22\,N_0}}\right)$

$\left(\text{NORMAL QPSK: } Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)\right)$

$\frac{\mathcal{E}_b}{N_0} \rightarrow \mathcal{E}_S = 2\mathcal{E}_b \Rightarrow P_e = Q\left(\sqrt{\frac{13 \cdot 2 \cdot \mathcal{E}_b}{22\,N_0}}\right) = Q\left(\sqrt{\frac{26\,\mathcal{E}_b}{22\,N_0}}\right) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0} \cdot \frac{13}{22}}\right)$

$\Longrightarrow$ SNR $\ll$ SNR(NORMAL QPSK) $\leftarrow$ QUASI METÀ e P_e più alta

$$P_{err} = \frac{1}{4}\left(P_e(S_1) + P_e(S_2) + P(S_3) + P(S_4) \right)$$

$$P_e(S_1) = 1 - P_c(S_1) = 1 - P(y_1(S_1) < 0)\cdot P(y_2(S_2) > 0)$$

$y_1(S_1) = -2a + n_1$ $\qquad \boxed{\frac{v^2}{N_0} = \frac{t^2}{2} \Rightarrow t = \sqrt{\frac{2}{N_0}}\,2a}$

$y_2(S_2) = a + n_2$

- $P(-2a + n_1 < 0) = P(n_1 < 2a) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}}\int_{-\infty}^{2a} e^{-\frac{v^2}{N_0}}\,dv = \frac{1}{\sqrt{\pi N_0}}\int_{-\infty}^{\sqrt{\frac{8a^2}{N_0}}} e^{-\frac{t^2}{2}}\sqrt{\frac{N_0}{2}}\,dt$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\sqrt{\frac{8a^2}{N_0}}} e^{-\frac{t^2}{2}}\,dt = 1 - Q\left(\sqrt{\frac{8a^2}{N_0}}\right) \Rightarrow \left(a = \frac{\sqrt{E_S}}{\sqrt{11}}\right) \Rightarrow 1 - Q\left(\sqrt{\frac{16 E_S}{11 N_0}}\right)$$

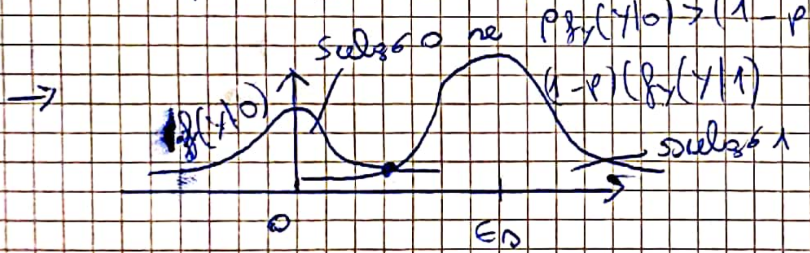
- $P(a + n_2 > 0)$ ← stessi calcoli, non svolto.

Esercizio 6 (Binary optical communication system)

$$E_S = \frac{A^2 T}{4}$$

$$\begin{cases} 0 & p & \sigma_n^2 = \sigma^2 E_S \\ 1 & 1-p & \sigma_n^2 = 2\sigma^2 E_S \end{cases}$$

simboli non equiprobabili → map criteria → $p f_Y(y|0) > (1-p) f_Y(y|1)$



1) Average energy per symbol?
2) probability of error?

$$E_{av} = p\cdot 0 + (1-p)E_S \iff p\left(\frac{1}{\sqrt{2\pi\sigma^2 E_S}} e^{-\frac{\lambda^2}{2\sigma^2 E_S}}\right) \gtrless (1-p)\left(\frac{1}{\sqrt{4\pi\sigma^2 E_S}} e^{-\frac{\lambda^2}{4\sigma^2 E_S}}\right)$$

esplicito λ = threshold

In case of normale ON-OFF keying → optimum threshold $= \frac{E_S}{2}$

→ Scegliamo il threshold come fossero equiprobabili → ma non lo sono, quindi calcoliamo le P_{err}.

$P_e = p P_e(0) + (1-p) P_e(1)$ $\qquad P_e(0) = P(n_1)$

Compio un errore quando $P\left(n_1 > \frac{E_S}{2}\right) = \frac{1}{\sqrt{2\pi\sigma^2 E_S}}\int_{\frac{E_S}{2}}^{+\infty} e^{-\frac{x^2}{2\sigma^2 E_S}}\,dx =$

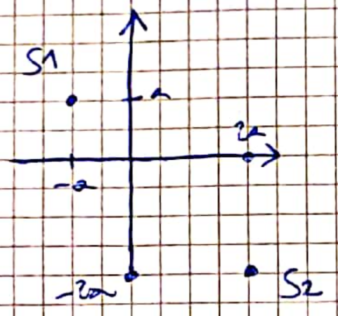
$$= \frac{1}{\sqrt{2\pi\sigma^2 E_S}}\int_{\sqrt{\frac{E_S}{4\sigma^2 E_S}}}^{+\infty} e^{-\frac{t^2}{2}}\sqrt{\sigma^2 E_S}\,dt = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{E_S}{4\sigma^2 E_S}}}^{+\infty} e^{-\frac{t^2}{2}}\,dt = Q\left(\sqrt{\frac{E_S}{4\sigma^2}}\right)$$

$P_e(1) = P\left(E_s + m_2 \le \frac{E_s}{2}\right) = P\left(m_2 < -\frac{E_s}{2}\right)$

Exercise (binary communication system passband)

AWGN $= \frac{N_0}{2}$

1) Average energy E_s?
2) Prob of error?



1) $E_{AV} = \frac{1}{2}\left(a^2 + a^2 + 4a^2 + 4a^2\right) \Rightarrow a = \sqrt{\frac{E_{AV}}{5}}$ (equiprobabile)

2) Normalmente faremmo $y_1 = -a + m_1$, $y_2 = a + m_2$

Se $y_1 < 0 \Rightarrow y_1$

Se $y_2 > 0 \Rightarrow y_2$

Qui invece facciamo $P_{err} = Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$

$d^2 = (2a+a)^2 + (-2a-a)^2 = 9a^2 + 9a^2 = 18a^2$

$(x_1 - x_2)^2 + (y_1 - y_2)^2$

$P_{err} = Q\left(\sqrt{\frac{9a^2}{N_0}}\right) = Q\left(\sqrt{\frac{9E_{AV}}{5N_0}}\right)$ IN THIS CASE $E_{AV} = E_b$ (AVERAGE)

Se consideriamo una BPSK con E_b uguali e questa è la $P_{error} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

$Q\left(\sqrt{\frac{9E_{AV}}{5N_0}}\right) > Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ poiché $\frac{9E_{AV}}{5N_0} < \frac{2E_b}{N_0}$

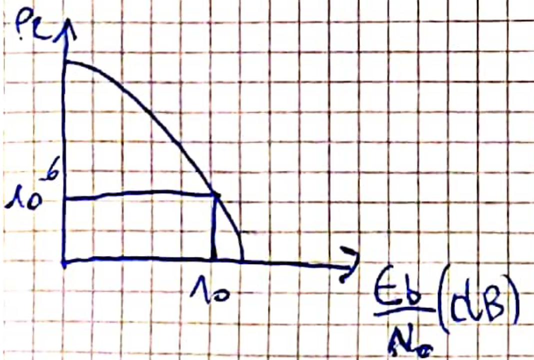
Dobbiamo moltiplicare per $x = \frac{10}{9}$ per ottenere col primo metodo le performance del secondo.

Esercizio (QPSK)

AWGN $\frac{N_0}{2} = 10^{-10}\ \frac{W}{Hz}$

$E_s = \frac{A^2 T}{2}$

$P_e \le 10^{-6}$



1) Trova A con $R_b = 10\ \frac{kbit}{s}$ (A), $100\ \frac{kbit}{s}$ (B), $1\ \frac{Mb}{s}$ (C)

$\left(\frac{E_b}{N_0}\right)_{linear} = 10^{\frac{10}{10}} = 10 \rightarrow$ IN THIS CASE IT'S THE SAME

(A) $R_b = 10\ \frac{kbit}{s} \Rightarrow T_b = 10^{-4}\ s$

$T_s = 2 \cdot T_b = 2 \cdot 10^{-4}\ s$

$E_b = \frac{E_s}{2} = \frac{A^2(2T_b)}{4} = \frac{A^2 T_b}{2}$

Ma io voglio che $\frac{E_b}{N_0} \ge 10 \Rightarrow \frac{A^2 T_b}{2N_0} = \frac{A^2 10^{-4}}{2N_0} \ge$

$\Rightarrow A = \sqrt{20} \cdot 10^{-2}$

(B) $R_b = 100\ \frac{kbit}{s} \Rightarrow$ Mi aspetto A maggiore

(C) $R_b = 1\ Mb/s \Rightarrow$ Mi aspetto A ancora maggiore

Esercizio 6 (M-QAM)

$B = 4\,kHz$

$R_b = 9600\,bps$

$\frac{N_0}{2} = 10^{-10}\,\frac{W}{Hz}$

Voglio fare M-QAM per avere $P_e \leq 10^{-6}$

ROLL OFF FACTOR $\alpha = 0{,}5$ (50%)

1) $R_s = \frac{R_b}{\log_2 M}$

$B = R_s(1+\alpha) \Rightarrow 4000 > \frac{9600}{\log_2 M}(1+0{,}5)$

$\Downarrow$

$\Rightarrow \log_2 M > \frac{9600}{4000}(1+0{,}5) \Rightarrow M \geq 16$

B MIN OCCUPIED BUT B DEVE ESSERE HIGHER SO I CHOOSE "$>$" AND NOT "$=$"

Potenza ?

$\rightarrow$ RICAVATO CON TABELLA

$\frac{P \cdot T_b}{N_0} = SNR = 10^{\frac{20}{10}} = 100 \longrightarrow$ RICAVA POTENZA